



Spreadsheet based thermal resistance network analysis



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This presentation presents an approach for filling the gap between simple one-dimensional thermal calculations and FEM/CFD analyses. Excel has substantial analysis capabilities and can also provide relatively simple user interfaces. The tutorial will discuss methods to estimate the thermal resistances within a system and how to automatically populate those values into a thermal resistance matrix. This thermal resistance matrix can then be solved using Excel matrix functions to determine system temperatures.

Why do thermal analysis using a spreadsheet?

- There are a lot of powerful software tools targeted at solving thermal problems. They are so simple that any fool can run them and produce believable looking results. So why think about using a spreadsheet?
- For starters, because any fool can run those tools and produce believable results...

Seriously, when should we use a spreadsheet?

- Spreadsheet-based analysis may be good for:
 - quickly identifying critical design factors
 - early designs that are not well enough defined to generate solid models
 - creating tools that can be shared with other people who don't use dedicated analysis tools such as ANSYS, Flotherm, etc.
 - analyzing simple situations so that you aren't tying up licenses of dedicated analysis tools
 - running a quick validation analysis to make sure that you didn't make a mistake with your normal analysis tool
- Spreadsheets won't:
 - replace analysis tools like Flotherm, ANSYS, etc.
 - be unaffected by fat finger mistakes

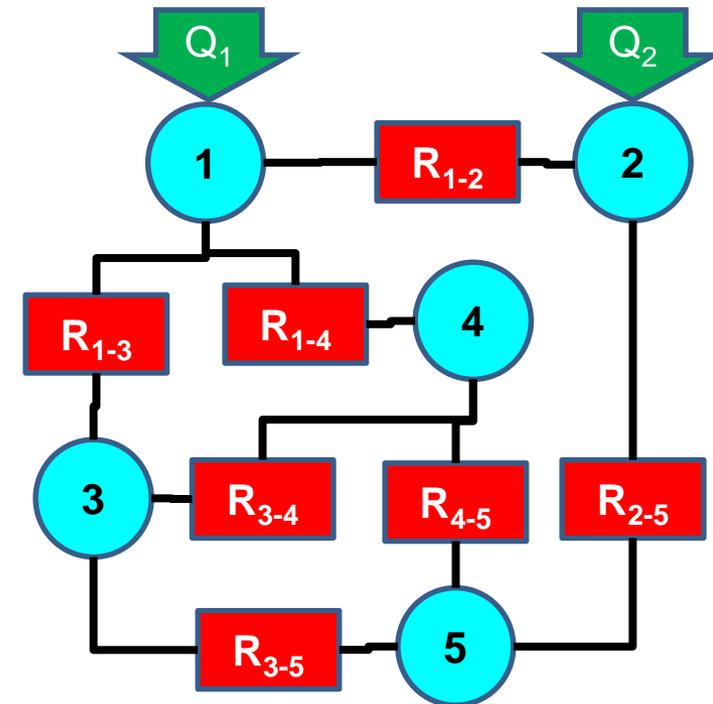
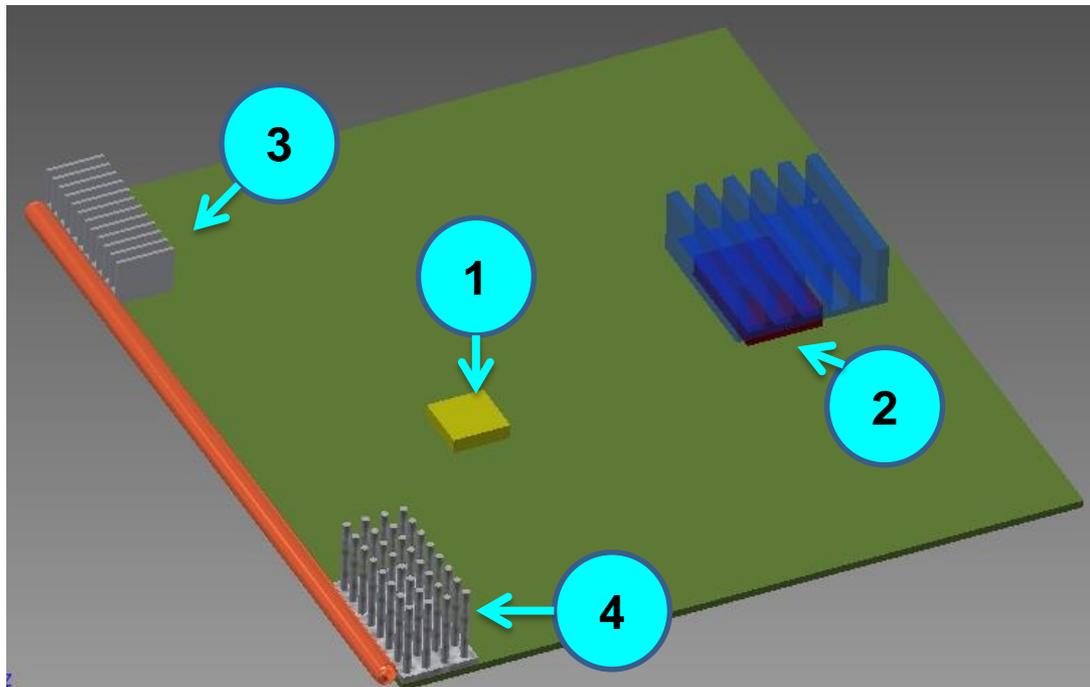
Resistance Network

- We can model a thermal system as a network of individual nodes that are connected with thermal resistances (R)
- For this analysis, we will only look at steady state thermal conditions, so we won't include thermal capacitance (mass*specific heat)
- Thermal resistance determines the heat transfer (Q) between two nodes at different temperatures (T):

$$Q_{a-b} = (T_a - T_b)/R_{a-b}$$

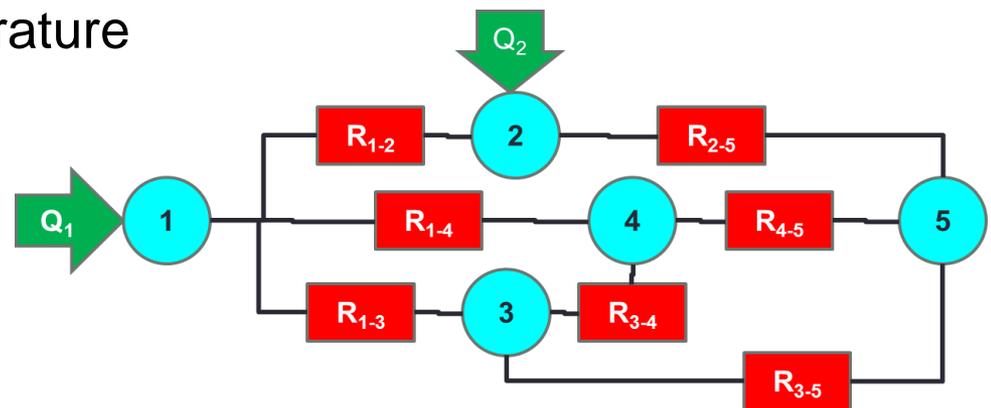
An Unrealistic Example of a Thermal Resistance Network

- Two heat dissipating components on a board with three heat sinks and a heat pipe
- Node 5 is the ambient air



Solving a Resistance Network

- Energy balance applied to each node: energy in + energy out = 0 (steady state conditions)
- T_5 is the ambient temperature, which stays the same, so we don't need to do an energy balance on it
 - node 1: $Q_1 - (T_1 - T_2) / R_{1-2} - (T_1 - T_3) / R_{1-3} - (T_1 - T_4) / R_{1-4} = 0$
 - node 2: $Q_2 + (T_1 - T_2) / R_{1-2} - (T_2 - T_5) / R_{2-5} = 0$
 - node 3: $(T_1 - T_3) / R_{1-3} - (T_3 - T_4) / R_{3-4} - (T_3 - T_5) / R_{3-5} = 0$
 - node 4: $(T_1 - T_4) / R_{1-4} + (T_3 - T_4) / R_{3-4} - (T_4 - T_5) / C_{4-5} = 0$
 - node 5: $T_5 = \text{ambient temperature}$



Rearranging Terms

- Rearranged equations and using conductance ($C = 1/R$) =>

- $(C_{1-2} + C_{1-3} + C_{1-4})T_1 - C_{1-2}T_2 - C_{1-3}T_3 - C_{1-4}T_4 = Q_1$

- $-C_{1-2}T_1 + (C_{1-2} + C_{2-5})T_2 = Q_2 + C_{2-5}T_5$

- $-C_{1-3}T_1 + (C_{1-3} + C_{3-4} + C_{3-5})T_3 - C_{3-4}T_4 = C_{3-5}T_5$

- $-C_{1-4}T_1 - C_{3-4}T_3 + (C_{1-4} + C_{3-4} + C_{4-5})T_4 = C_{4-5}T_5$

- In matrix notation, this is $[C]\{T\} = [Q]$

$$\begin{bmatrix} (C_{1-2} + C_{1-3} + C_{1-4}) & -C_{1-2} & -C_{1-3} & -C_{1-4} \\ -C_{1-2} & (C_{1-2} + C_{2-5}) & 0 & 0 \\ -C_{1-3} & 0 & (C_{1-3} + C_{3-4} + C_{3-5}) & -C_{3-4} \\ -C_{1-4} & 0 & -C_{3-4} & (C_{1-4} + C_{3-4} + C_{4-5}) \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{Bmatrix} = \begin{bmatrix} Q_1 \\ Q_2 + C_{2-5}T_5 \\ C_{3-5}T_5 \\ C_{4-5}T_5 \end{bmatrix}$$

- Solve for each temperature using matrix algebra:

- $\{T\} = [C]^{-1}[Q]$

Three steps to use a Resistance Network

- Set up the network
 - define nodes
 - estimate thermal resistances between each node
- Convert the individual resistances and boundary conditions into matrices
 - the conductance matrix, $[C]$, comes from the resistances
 - the boundary condition matrix, $[Q]$, comes from the heat loads and the ambient temperature (and resistances that are connected to ambient)
- Apply matrix algebra to solve for temperatures

Focus of this Presentation

Step 1: Defining Thermal Resistances

- Real systems are inherently 3-dimensional; thermal resistances are based on an assumption of 1-dimensional heat flow
- Types of thermal resistances
 - 1-D conduction; $R = L/kA$ for planar, $R = \ln(r_2/r_1)/2\pi Lk$ for radial, etc.
 - uniform surface temperature convection ($R = 1/hA$)
 - interface resistance due to TIMs, contact resistance, etc. ($R = R^*/A$)
 - R^* = thermal impedance, such as $C \text{ cm}^2/W$
 - component resistance (θ_{j-a} , θ_{j-b} , etc.)
 - various methods to treat 2-D heat flow as 1-D
 - spreading resistance, 'cooling circle', etc.
- For this presentation, I am assuming that you can come up with the thermal resistance values that you need
 - spreadsheets do give you some flexibility for implementing these and modifying them by only changing a cell or two
 - you can create generic resistance tools on self contained worksheets

Step 2: Generating the Conductance and Boundary Condition Matrices

- There are boatloads of ways to do this; I am just suggesting my way of doing it
- The goal here isn't elegance or computational efficiency
 - the goal is to make the best use of the capabilities of spreadsheet in order to make our lives easier!
- So, how do we let the spreadsheet do all the work to convert lists of thermal resistances and power inputs into the [C] and [Q] matrices?
 - first generate a resistance matrix
 - convert values in the resistance matrix to $1/R$ values
 - then generate a conductance matrix

Generic Resistance Network Solver

Resistance	Value	Node	Q	Temp
R_1-2	3	1	9	66.5
R_1-3	20	2	4	49.6
R_1-4	9	3	4	42.4
R_2-5	1	4	5	47.0
R_3-4	4	5	7	40.0
R_3-5	1	6	7	
R_4-5	7	7	8	
R_5-10	1.00E-06	8	9	
		T_10	40	

Resistance										
	1	2	3	4	5	6	7	8	9	10
1	#N/A	3	20	9	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A
2	3	#N/A	#N/A	#N/A	1	#N/A	#N/A	#N/A	#N/A	#N/A
3	20	#N/A	#N/A	4	1	#N/A	#N/A	#N/A	#N/A	#N/A
4	9	#N/A	4	#N/A	7	#N/A	#N/A	#N/A	#N/A	#N/A
5	#N/A	1	1	7	#N/A	#N/A	#N/A	#N/A	#N/A	1E-06
6	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A
7	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A
8	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A
9	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A
10	#N/A	#N/A	#N/A	#N/A	1E-06	#N/A	#N/A	#N/A	#N/A	#N/A

Individual Conductances (1/R)										
	1	2	3	4	5	6	7	8	9	10
1	0	0.333333	0.05	0.111111	0	0	0	0	0	0
2	0.333333	0	0	0	1	0	0	0	0	0
3	0.05	0	0	0.25	1	0	0	0	0	0
4	0.111111	0	0.25	0	0.142857	0	0	0	0	0
5	0	1	1	0.142857	0	0	0	0	0	1000000
6	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	1000000	0	0	0	0	0

Conductance Matrix										
	1	2	3	4	5	6	7	8	9	10
1	0.494444	-0.333333	-0.05	-0.111111	0	0	0	0	0	0
2	-0.333333	1.333333	0	0	-1	0	0	0	0	0
3	-0.05	0	1.3	-0.25	-1	0	0	0	0	0
4	-0.111111	0	-0.25	0.50297	-0.1429	0	0	0	0	0
5	0	-1	-1	-0.1429	1000002	0	0	0	0	-1E+06
6	0	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	0	1	0	0	0
8	0	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	0	1	0
10	0	0	0	0	-1E+06	0	0	0	0	1000000

Resistance Matrix

1/R Matrix

Input Thermal Resistances

Input Thermal Resistances

Conductance Matrix

BC Matrix

BC Matrix

Resistance Matrix

- With the resistance values in two columns with consistent formatting, you can generate a resistance matrix
- Start with an NxN array with numbers from 1 to N along the top and the left edge (N is the maximum # of nodes)

```
=IF($J4<K$3,OFFSET($J$3,K$3,$J4),VLOOKUP("R_"&K$3&"-
"&$J4,$B$4:$C$48,2,FALSE))
```

- Populates cells with the resistance values defined in the input region
- #N/A if no value available

		Resistances									
		1	2	3	4	5	6	7	8	9	10
1	#N/A	3	20	9	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A
2	3	#N/A	#N/A	#N/A	1	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A
3	20	#N/A	#N/A	4	1	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A
4	9	#N/A	4	#N/A	7	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A
5	#N/A	1	1	7	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	1E-06
6	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A
7	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A
8	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A
9	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A
10	#N/A	#N/A	#N/A	#N/A	#N/A	1E-06	#N/A	#N/A	#N/A	#N/A	#N/A

Converting Inputs into the [C] and [Q] Matrices

- The [C] matrix that we saw before followed a pattern:
 - terms in each cell is $-1/R$, except along the diagonal (row = column) the term is the sum of $1/R$ for that row (or column)

$$\begin{bmatrix} (C_{1-2} + C_{1-3} + C_{1-4}) & -C_{1-2} & -C_{1-3} & -C_{1-4} \\ -C_{1-2} & (C_{1-2} + C_{2-5}) & 0 & 0 \\ -C_{1-3} & 0 & (C_{1-3} + C_{3-4} + C_{3-5}) & -C_{3-4} \\ -C_{1-4} & 0 & -C_{3-4} & (C_{1-4} + C_{3-4} + C_{4-5}) \end{bmatrix}$$

- Terms in the [Q] matrix are the heat dissipation plus the conductance * ambient temperature

$$\begin{bmatrix} Q_1 \\ Q_2 + C_{2-5}T_5 \\ C_{3-5}T_5 \\ C_{4-5}T_5 \end{bmatrix}$$

1/R and Conductance Matrices

`=IF(ISERROR(K4),0,1/K4)`

If the value in the R matrix is #N/A, value is 0, if not, value is 1/R

adds up the 1/R values in the same column; if the sum = 0, set the value to 1

`=IF(SUM(K19:K28)=0,1,SUM(K19:K28))`

`=IF(K$34=$J35,K$30,-K19)`

if the row and column numbers are the same, use the sum term for the column, otherwise use the -1/R term

	Individual Conductances (1/R)									
	1	2	3	4	5	6	7	8	9	10
1	0	0.333333	0.05	0.111111	0	0	0	0	0	0
2	0.333333	0	0	0	1	0	0	0	0	0
3	0.05	0	0	0.25	1	0	0	0	0	0
4	0.111111	0	0.25	0	0.14286	0	0	0	0	0
5	0	1	1	0.14286	0	0	0	0	0	1000000
6	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	1000000	0	0	0	0	0
	0.49444	1.33333	1.3	0.50397	1000002	1	1	1	1	1000000
	Influence Matrix									
	1	2	3	4	5	6	7	8	9	10
1	0.49444	-0.33333	-0.05	-0.11111	0	0	0	0	0	0
2	-0.33333	1.33333	0	0	-1	0	0	0	0	0
3	-0.05	0	1.3	-0.25	-1	0	0	0	0	0
4	-0.11111	0	-0.25	0.50397	-0.14286	0	0	0	0	0
5	0	-1	-1	-0.14286	1000002	0	0	0	0	-1000000
6	0	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	0	1	0	0	0
8	0	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	0	1	0
10	0	0	0	0	-1000000	0	0	0	0	1000000

Step 3: Matrix Analysis to Solve for Temps

- Excel has functions for dealing with matrices
 - Minverse() calculates the inverse of a matrix
 - Mmult() multiplies to matrices
- To use, highlight entire region where result will go, type in equation and then hit ctrl-shift-enter
 - resulting matrix equation will have curly brackets (you don't put in)

{=MINVERSE(K35:S43)}

calculates the inverse of [C]

{=MMULT(K50:S58,W35:W43)}

calculates the inverse of [C]⁻¹[Q]

Inverse Conductance Matrix									
	1	2	3	4	5	6	7	8	9
1	2.650988	0.662748	0.236966	0.70202	0.000001	0	0	0	0
2	0.662748	0.915688	0.059242	0.175506	0.000001	0	0	0	0
3	0.236966	0.059242	0.871534	0.484581	0.000001	0	0	0	0
4	0.70202	0.175506	0.484581	2.379411	0.000001	0	0	0	0
5	0.000001	0.000001	0.000001	0.000001	0.000001	0	0	0	0
6	0	0	0	0	0	1	0	0	0
7	0	0	0	0	0	0	1	0	0
8	0	0	0	0	0	0	0	1	0
9	0	0	0	0	0	0	0	0	1

Node	Temp
1	66.5
2	49.6
3	42.4
4	47.0
5	40.0
6	0.0
7	0.0
8	0.0
9	0.0

General things for generic solver

- The spreadsheet was set up with node 10 defined as the ambient temperature
 - to make things work for my example and maintain the same node notations, I connected node 5 to node 10 with a very, very low thermal resistance
- I used Data Validation to limit what names I could give the resistances (so that I couldn't type a name in incorrectly)*
- I used Conditional Formatting to hide the temperatures calculated for inactive nodes*

**see back up information for details*

Spreadsheet Setup

- It helps if you can treat different solution steps as separate subroutines, with each subroutine being on a different worksheet
- One worksheet is where you input geometry and other conditions
- Other worksheets use those inputs to calculate individual thermal resistance values
- The generic matrix solver is just another subroutine / worksheet that you can reuse in other analyses

Summary

- Spreadsheets are handy for doing quick analysis – particularly if you want to share the analysis tool with other users
- Break analysis into separate, reusable ‘sub-routines’ that consist of self-contained worksheets
 - a matrix solver is just another of these sub-routines
- Spreadsheet based thermal resistance analysis can help the gap between ‘back of the envelope’ initial feasibility calculations and detailed FEM/CFD optimization/design verification

SPREADSHEET EXAMPLE

THERMAL LIVE 2015

Thanks for attending!

Don't miss Thermal Live 2016!

Fall 2016

www.thermallive2016.com

Back up Information

Generating a Resistance Matrix

- Start with a list of all of your resistances in your system
- Key is to use a standard naming convention
 - use something like R_1-2 for the resistance between nodes 1 and 2
 - be consistent! if you use “R_” for one, use them for all
 - no need to define R_2-1, because it will be the same as R_1-2
 - but you must be consistent and show the smaller number first (or the larger – just be consistent!)
- Create a list of your resistances with the names in one column and the values for each resistance in the adjacent column
 - when I use the term ‘name’ here, I am being informal; DO NOT explicitly name the values using the ‘Name Manager’ on the “Formulas” tab

Data Entry Format for Resistances

- Create your list of thermal resistance values in two adjacent columns with the left column showing the name
 - name needs to include the numbers (or letters – however you define the nodes) for each node
- Ideally, the values of the resistances are calculated elsewhere in your spreadsheet based on resistance equations
 - (you know, those ones that I said I wouldn't talk about...)
- Likewise, generate a list of power dissipations for each node

Resistance	Value	Node	Q
R_1-2	3	1	9
R_1-3	20	2	4
R_1-4	9	3	
R_2-5	1	4	
R_3-4	4	5	
R_3-5	1	6	
R_4-5	7	7	
R_5-10	1.00E-06	8	
		9	
		T_10	40

To help ensure that you use the right format for the resistance name, you might use Data Validation (see appendix)

Data Validation

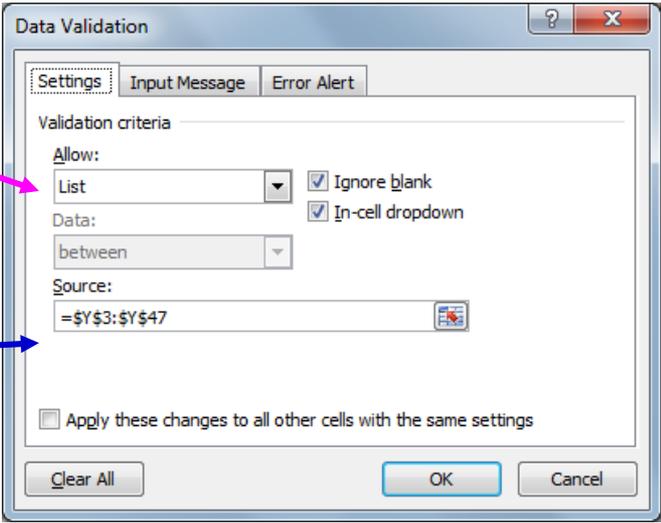
- To start, make a list of all the possible resistance names to choose from
- Highlight the cells in which you would enter the resistance name
- On the Data tab, select Data Validation / Data Validation

X	Y
	R_1-2
	R_1-3
	R_1-4
	R_1-5
	R_1-6
	R_1-7
	R_1-8
	R_1-9
	R_1-10
	R_2-3

R_1-2
R_1-3
R_1-4
R_1-5
R_1-6
R_1-7
R_1-8
R_1-9
R_1-10
R_2-3
R_2-4
R_2-5
R_2-6
R_2-7
R_2-8
R_2-9
R_2-10
R_3-4
R_3-5
R_3-6
R_3-7
R_3-8
R_3-9
R_3-10
R_4-5
R_4-6
R_4-7
R_4-8
R_4-9
R_4-10
R_5-6
R_5-7
R_5-8
R_5-9
R_5-10
R_6-7
R_6-8
R_6-9
R_6-10
R_7-8
R_7-9
R_7-10
R_8-9
R_8-10
R_9-10

select 'List'

select range where the predefined list of names is



Resistance	Value
R_1-2	3
R_1-3	20
R_1-4	9
R_2-5	1
R_3-4	4
R_3-5	1
R_4-5	7
R_5-10	1.00E-06

Details on Function for R Matrix

```
=IF($J4<K$3,OFFSET($J$3,K$3,$J4),VLOOKUP("R_"&K$3&"-
"&$J4,$B$4:$C$48,2,FALSE))
```

Visual Basic	What it Does?	Why it Does it?
=IF(\$J4<K\$3,	If the row # is less than the column #, then...	<p>if a cell is below the diagonal of the matrix, this will grab the corresponding value in the symmetric matrix (i.e, says that $R_{2-1} = R_{1-2}$)</p> <p>If the cell is above or on the diagonal, its value will be equal to thermal resistance (in the list) for R_{a-b}, where a is the column # and b is the row number.</p> <p>If no value is found, the vlookup() function returns #N/A</p>
OFFSET(\$J\$3,K\$3,\$J4),	look at the value in the cell in which the row & column #'s are switched	
VLOOKUP(looks up up a value in in the same row of an array	
R_"&K\$3&"-"&\$J4,	generates a text block that consists of R_column #-row #	
\$B\$4:\$C\$48,	defines what array to query	
2,	defines which column in the array to get the data from	
FALSE))	says that names in the 1 st column may not be in order	

Conditional Formatting

- Highlight over a cell in the (yellow) temperature output array
- On the Home tab, select Conditional Formatting / New Rule
- Select 'Use a formula to determine which cells to format'
- in the Rule Description, enter $=(\text{cell ref}) = 0$, where cell ref is the location of your cell (ex. H4)
- under Format, change the font color to match the yellow background

Resistance	Value	Node	Q	Temp
R_1-2	3	1	9	66.5
R_1-3	20	2	4	49.6
R_1-4	9	3		42.4
R_2-5	1	4		47.0
R_3-4	4	5		40.0
R_3-5	1	6		0.0
R_4-5	7	7		0.0
R_5-10	1.00E-06	8		0.0
		9		0.0